



中國地質大學  
CHINA UNIVERSITY OF GEOSCIENCES  
1952-2022  
七十周年校庆  
THE 70TH ANNIVERSARY

**2022 International Symposium on Differential Equations**

# Academic Program

May 14-16, 2022

Center for Mathematical Sciences & School of Mathematics and Physics

China University of Geosciences

Wuhan · China



## 2022 International Symposium on Differential Equations

### Overview

The 2022 International Symposium on Differential Equations will be held as a full-online format hosted at China University of Geosciences (Wuhan), May 14th-16th, 2022. It will provide a forum for exchanges of the latest results related to differential equations. It will also provide an excellent opportunity for young researchers to interact with our scientists and learn hands-on research experience in these fields. All the talks will be held online using Tencent Meeting platform.

### Organizing Committee

Jianhong Wu (York University)

Shangjiang Guo (China University of Geosciences)

### Contact Information

Dr Zhisu Liu

China University of Geosciences, liuzhisu@cug.edu.cn, 15211363659

Dr Shangzhi Li

China University of Geosciences, lishangzhi@cug.edu.cn, 18075191139

Dr Shangjiang Guo

China University of Geosciences, guosj@cug.edu.cn, 027-67883093

### Sponsored by

Center for Mathematical Sciences, China University of Geosciences

School of Mathematics and Physics, China University of Geosciences

### Invited Speakers and Chairs

**Yuming Chen**, Wilfrid Laurier University, ychen@wlu.ca

**Yihong Du**, University of New England, ydu@une.edu.au

**Meng Fan**, Northeast Normal University, mfan@nenu.edu.cn

**Mao-An Han**, Shanghai Normal University, mahan@shnu.edu.cn

**Lihong Huang**, Hunan University & Changsha University, lhhuang@csust.edu.cn

**Wen Huang**, University of Science and Technology of China, wenh@mail.ustc.edu.cn

**Jibin Li**, Zhejiang Normal University, lij@zjnu.cn

**Xing Liang**, University of Science and Technology of China, xliang@ustc.edu.cn

**Rongsong Liu**, University of Wyoming, Rliu1@uwyo.edu

**Yijun Lou**, Hong Kong Polytechnic University, yijun.lou@polyu.edu.hk

**Yuan Lou**, Shanghai Jiaotong University, yuanlou@sjtu.edu.cn

**Kening Lu**, Brigham Young University, klu@math.byu.edu

**Chunhua Ou**, Memorial University of Newfoundland, ou@mun.ca

**Shigui Ruan**, University of Miami, ruan@math.miami.edu  
**Song Shao**, University of Science and Technology of China, songshao@ustc.edu.cn  
**Wenxian Shen**, Auburn University, wenxish@auburn.edu  
**Yi Wang**, University of Science and Technology of China, wangyi@ustc.edu.cn  
**Zhi-An Wang**, Hong Kong Polytechnic University, mawza@polyu.edu.hk  
**Juncheng Wei**, University of British Columbia, jcwei@math.ubc.ca  
**Dongmei Xiao**, Shanghai Jiaotong University, xiaodm@sjtu.edu.cn  
**Yingfei Yi**, University of Alberta and Jilin University, yingfei@ualberta.ca  
**Jianshe Yu**, Guangzhou University, jsyu@gzhu.edu.cn  
**Rong Yuan**, Beijing Normal University, ryuan@bnu.edu.cn  
**Yuan Yuan**, Memorial University of Newfoundland, yyuan@mun.ca  
**Xiang Zhang**, Shanghai Jiaotong University, xzhang@sjtu.edu.cn  
**Xiaoqiang Zhao**, Memorial University of Newfoundland, zhao@mun.ca  
**Huaiping Zhu**, York University, huaiping@yorku.ca  
**Xingfu Zou**, Western University, xzou@uwo.ca

### Tencent Meeting platform Information

会议主题：微分方程国际会议(The International Symposium on Differential Equations)  
会议时间：2022/05/14 07:30-23:30 (GMT+08:00) 中国标准时间 - 北京  
重复周期：2022/05/14-2022/05/16 07:30-23:30, 每天

点击链接入会，或添加至会议列表：

 <https://meeting.tencent.com/dm/dhMxH7MoVlw>

#腾讯会议：470-9369-4496

手机一键拨号入会

+8675536550000,,47093694496# (中国大陆)

+85230018898,,,2,47093694496# (中国香港)

根据您的位置拨号

+8675536550000 (中国大陆)

+85230018898 (中国香港)

复制该信息，打开手机腾讯会议即可参与

**Program**

**May 14 (Saturday), 2022 (Beijing time)**  
**Tencentmeeting Room ID: 470-9369-4496**

|              |                       |  |
|--------------|-----------------------|--|
| <b>Chair</b> | <b>Shangjiang Guo</b> |  |
| <b>Time</b>  | <b>Speaker</b>        | <b>Opening ceremony</b>  |
| 8:10-8:20    | <b>Yongsheng Liu</b>  |  |
| 8:20-8:30    | <b>Jianhong Wu</b>    |  |
| <b>Chair</b> | <b>Jianhong Wu</b>    |  |
| <b>Time</b>  | <b>Speaker</b>        | <b>Title</b>   |
| 8:30-9:10    | <b>Kening Lu</b>      | Ergodicity, mixing, limit theorems for quasi-periodically forced 2D stochastic Navier-Stokes Equations |
| 9:10-9:50    | <b>Juncheng Wei</b>   | Gross-Pitaeskkii equation, Adler-Moser polynomials and Kadomtsev-Petviashvili lump                     |
| 9:50-10:00   | Coffee break          |  |
| <b>Chair</b> | <b>Lihong Huang</b>   |  |
| <b>Time</b>  | <b>Speaker</b>        | <b>Title</b>   |
| 10:00-10:40  | <b>Yuming Chen</b>    | Some notes on Lyapunov's direct method   |
| 10:40-11:20  | <b>Zhi-An Wang</b>    | 具有资源依赖性扩散的种群动力学：<br>单物种和双物种模型  |
| 11:20-12:00  | <b>Rongsong Liu</b>   | Multiple dose pharmacokinetic models predict bioavailability of toxins in vertebrate herbivores        |
| 12:00-14:30  | <b>Lunchtime</b>      |  |

| <b>Chair</b> |                       | <b>Mao-An Han</b>  |  |
|--------------|-----------------------|--|--|
| <b>Time</b>  | <b>Speaker</b>        | <b>Title</b>   |  |
| 14:30-15:10  | <b>Jianshe Yu</b>     | Global asymptotic stability in a delay mosquito population suppression model   |  |
| 15:10-15:50  | <b>Dongmei Xiao</b>   | Planar polynomial differential systems with a global center  |  |
| 15:50-16:00  | Coffee break          |  |  |
| <b>Chair</b> |                       | <b>Jibin Li</b>  |  |
| <b>Time</b>  | <b>Speaker</b>        | <b>Title</b>   |  |
| 16:00-16:40  | <b>Yihong Du</b>      | Spreading profile of two invading competitors  |  |
| 16:40-17:20  | <b>Yi Wang</b>        | $C^1$ -theory for smooth non-autonomous monotone dynamical systems   |  |
| 17:20-19:00  | Dinnertime            |  |  |
| <b>Chair</b> |                       | <b>Zhisu Liu</b>   |  |
| <b>Time</b>  | <b>Speaker</b>        | <b>Title</b>   |  |
| 19:30-20:10  | <b>Xiaoqiang Zhao</b> | Propagation dynamics of reaction and diffusion equations in a time-heterogeneous shifting environment                  |  |
| 20:10-20:50  | <b>Yuan Yuan</b>      | Nonlinear dynamics in modeling sea lice with stage structure   |  |
| 20:50-21:30  | <b>Chunhua Ou</b>     | Existence, uniqueness and stability of forced waves to the Lotka-Volterra competition system in a shifting environment |  |

**May 15 (Sunday), 2022 (Beijing time)**  
**Tencentmeeting Room ID: 470-9369-4496**

|              |                     |   |
|--------------|---------------------|---|
| <b>Chair</b> | <b>Rong Yuan</b>    |   |
| Time         | Speaker             | Title   |
| 8:30-9:10    | <b>Wenxian Shen</b> | Population dynamics under climate change: persistence criterion and effects of fluctuations   |
| 9:10-9:50    | <b>Shigui Ruan</b>  | Age-structured population dynamics with nonlocal diffusion                                    |
| 9:50-10:00   | Coffee break        |   |
| <b>Chair</b> | <b>Xing Liang</b>   |   |
| Time         | Speaker             | Title   |
| 10:00-10:40  | <b>Xingfu Zou</b>   | Evolution and adaptation of anti-predation response of prey in a two-patchy environment       |
| 10:40-11:20  | <b>Huaiping Zhu</b> | Bifurcation of nilpotent singularities and Hilbert's 16th problem for quadratic vector fields |
| 11:20-12:00  | <b>Yijun Lou</b>    | A delayed succession model with diffusion for the impact of diapause on population growth     |
| 12:00-14:30  | <b>Lunchtime</b>    |   |
| <b>Chair</b> | <b>Meng Fan</b>     |   |
| Time         | Speaker             | Title   |
| 14:30-15:10  | <b>Xiang Zhang</b>  | Predator-Prey models with Sigmoid functional response   |
| 15:10-15:50  | <b>Yuan Lou</b>     | Coexistence of strains in some diffusive epidemic models                                      |
| 15:50-16:00  | Coffee break        |   |
| <b>Chair</b> | <b>Song Shao</b>    |   |
| Time         | Speaker             | Title   |
| 16:00-16:40  | <b>Yingfei Yi</b>   | Quasi-stationary distributions  |
| 16:40-17:20  | <b>Wen Huang</b>    | Entropy, Horseshoe and Katok conjecture   |

Titles and abstracts (in alphabetical order)

## **Some notes on Lyapunov's direct method**

**Yuming Chen**

Wilfrid Laurier University

ychen@wlu.ca

The global stability of equilibria of epidemic models plays an important role in understanding the transmission mechanism of the diseases. One of the most powerful approaches to determine the global stability is Lyapunov's direct method. The method involves constructing an appropriate Lyapunov function and verifying the negative (semi-)definiteness of its derivative along solutions, which often are not easy to achieve. This talk reports our recent progress in applying Lyapunov's direct method. On the one hand, we provide a new kind of function for constructing Lyapunov functions. On the other hand, when the form of a Lyapunov function is given, we develop some novel arguments to establish the negative (semi-)definiteness of its derivative along solutions.

## **Spreading profile of two invading competitors**

**Yihong Du**

University of New England

ydu@une.edu.au

In this talk I will report some recent advances on the research of a two species competition model with free boundaries. We are interested in a complete understanding of the spreading profiles of the system in the weak-strong competition case. We show that, as the initial state varies, there are exactly five different types of long-time dynamical behavior for this system. The talk is based on theoretical work with Chang-Hong Wu, and numerical work with K. Khan, Shuang Liu and T. Schaerf.

## **Entropy, Horseshoe and Katok conjecture**

**Wen Huang**

University of Science and Technology of China

wenh@mail.ustc.edu.cn

In this talk, we will introduce weak Horseshoe and semi-horseshoe. We will review some progress about weak horseshoe, semi-Horseshoe and Katok conjecture. Particularly,

- For random compact set, Positive (fiber) entropy implies (fiber) weak Horseshoe.
- The existence of semi-horseshoes for partially hyperbolic diffeomorphisms.
- Katok conjecture holds for an affine transformation on a compact homogeneous space.

This based on joint works with Prof. Li, Lu, Xu, Ye

## **Multiple dose pharmacokinetic models predict bioavailability of toxins in vertebrate herbivores**

**Rongsong Liu**

University of Wyoming

Rliu1@uwyo.edu

Compartmental pharmacokinetic models are built to predict the concentration of toxic phytochemical in the gastrointestinal tract and blood following oral intake by an individual vertebrate herbivore. The existing single and multiple dose pharmacokinetic models are extended by inclusion of impulsive differential equations which account for an excretion factor whereby unchanged toxins are excreted in the feces due to gastrointestinal mobility. An index  $\alpha$  is defined to measure the fraction of bioavailability attributed to the excretion factor of gastrointestinal motility. Sensitivity analysis was conducted and suggests, for any toxin, the bioavailability index  $\alpha$  depends mostly on absorption rate of toxin from gastrointestinal tract into the blood, frequency of elimination due to gastrointestinal motility, and the frequency of toxin intake, under the model assumptions.

## **A delayed succession model with diffusion for the impact of diapause on population growth**

**Yijun Lou**

Hong Kong Polytechnic University

yijun.lou@polyu.edu.hk

Diapause, a period of arrested development driven by adverse environmental conditions, plays an important role in the establishment and invasion of insects and other invertebrate organisms in temperate and subtropical areas. In order to describe the spatial dynamics of diapausing species, this talk will present a novel model involving (a) seasonal succession to distinguish the normal growth period, diapause period, and post diapause period; (b) diffusion term to represent the random movement of species; and (c) maturation delay term to describe the developmental duration of species. The model in a bounded spatial domain will be investigated for the survival and establishment of a species. The extinction and persistence of the species can be predicted by the basic reproduction ratio. The model in an unbounded spatial domain will be



studied for the spreading of the species. The analytical results show that the minimal wave speed for periodic traveling wave is equal to the spreading speed. Numerical simulations are performed to validate theoretical results, and in particular to compare the effects of two diapausing strategies, diapausing in the adult stage and in the immature stage. This talk is based on a joint work with Drs. Zhenguo Bai and Xiao-Qiang Zhao.

## **Coexistence of strains in some diffusive epidemic models**

**Yuan Lou**

Shanghai Jiaotong University

yuanlou@sjtu.edu.cn

We study the dynamics of some multi-strain reaction-diffusion epidemic model and investigate how the coexistence of strains is impacted by the movement of populations and the environmental heterogeneity. Sufficient conditions are given to ensure the extinction of strains. For the case of two strains, general conditions for the existence, uniqueness and stability of coexistence endemic equilibrium are found. Surprisingly, when there is no coexistence of strains, it is possible for the "weak" strain to be dominant for intermediate diffusion rates, in strong contrast to small and large diffusion cases where the "weak" strain goes extinct. Finally, the asymptotic behaviors of the coexistence endemic equilibrium are investigated, where the spatial segregation of two strains is revealed. This is based on joint works with Rachidi Salako.

## **Ergodicity, mixing, limit theorems for quasi-periodically forced 2D stochastic Navier-Stokes Equations**

**Kening Lu**

Brigham Young University

klu@math.byu.edu

We consider the incompressible 2D Navier-Stokes equations on the torus driven by a deterministic time quasi-periodic force and a noise that is white in time and extremely degenerate in Fourier space. We show that the asymptotic statistical behavior is characterized by a uniquely ergodic and exponentially mixing quasi-periodic invariant measure. The result is true for any value of the viscosity  $\nu > 0$ . By utilizing this quasi-periodic invariant measure, we show the strong law of large numbers and central limit theorem for the continuous time inhomogeneous solution processes. Estimates of the corresponding rate of convergence are also obtained, which is the same as in the time homogeneous case for the strong law of large numbers, while the

convergence rate in the central limit theorem depends on the Diophantine approximation property on the quasi-periodic frequency and the mixing rate of the quasi-periodic invariant measure. We also prove the existence of a stable quasi-periodic solution in the laminar case (when the viscosity is large). This talk is based on a joint work with Liu Rongchang.

## **Existence, uniqueness and stability of forced waves to the Lotka-Volterra competition system in a shifting environment**

**Chunhua Ou**

Memorial University of Newfoundland  
ou@mun.ca

We study the existence, uniqueness as well as the stability of forced traveling waves for the Lotka-Volterra competition system in a shifting habitat. Based on the asymptotic behaviors and by means of upper-lower solution method coupled with sliding technique, we show that the forced wave for the system exists and is unique, when the forced speed lies in a specific interval. The explicit expressions of the two end points of this interval are derived and we discover that they are related to the Fisher-KPP-type invasion speed. Furthermore, we establish a squeezing theorem to show the local stability of the forced traveling waves. Finally, with the aid of comparison principle, we establish the global stability of the forced traveling wave fronts in a weighted functional space when the initial perturbation around the forced traveling wave fronts decays exponentially as  $x$  tends to infinity.

## **Age-structured population dynamics with nonlocal diffusion**

**Shigui Ruan**

University of Miami  
ruan@math.miami.edu

Individuals need to be mature enough to disperse and recent observations demonstrate that nonlocal diffusion processes are more applicable to many biological and epidemiological problems compared with random diffusion processes. Consequently it is more reasonable to study age-structured population models with nonlocal (convolution) diffusion. In this talk, I will introduce some recent results on the basic theory for age-structured population dynamics with nonlocal diffusion (Based on joint studies with Hao Kang and Xiao Yu).

## **Population dynamics under climate change: persistence criterion and effects of fluctuations**

**Wenxian Shen**

Auburn University

wenxish@auburn.edu

The current talk is concerned with population dynamics under climate change. The evolution of species is modelled by a reaction-diffusion equation in a spatio-temporally heterogeneous environment described by a climate envelope that shifts with a time -dependent speed function. For a general almost-periodic speed function, we establish the persistence criterion in terms of the sign of the approximate top Lyapunov exponent and, in the case of persistence, prove the existence of a unique forced wave solution that dominates the population profile of species in the long run. In the setting for studying the effects of fluctuations in the shifting speed or location of the climate envelope, we show by means of matched asymptotic expansions and numerical simulations that the approximate top Lyapunov exponent is a decreasing function with respect to the amplitude of fluctuations, yielding that fluctuations in the shifting speed or location have negative impacts on the persistence of species. In addition, we assert that large fluctuations can always drive a species to extinction.

## **$C^1$ -theory for smooth non-autonomous monotone dynamical systems**

**Yi Wang**

University of Science and Technology of China

wangyi@ustc.edu.cn

In this talk, we will report the  $C^1$ -theory and its recent progress on smooth non-autonomous monotone dynamical systems. This talk is based on a series of joint works with Jinxiang Yao.

## **具有资源依赖性扩散的种群动力学：单物种和双物种模型**

**Zhi-An Wang**

Hong Kong Polytechnic University

mawza@polyu.edu.hk

我们将讨论具有资源依赖性分散的单一物种和具有竞争的两个物种的种群模型。对于单物种模型，众所周知，如果扩散是简单的随机扩散，则由环境支持的总种群总是大于环境承载能力。然而，我们发现当分散依赖于资源分布时，支持的总人口可以等于或小于环境承载能力。这一分析结果不仅与酵母实验观察结果吻合，而且表明资源依赖型扩散与随机扩散相比可能会产生不同的结果。对于双物种竞争模型，当两个竞争物种使用相同的扩散策略相似（即它们的扩散策略成比

例) 或两个扩散系数都很大时, 我们给出了全局动力学的分类。我们还通过例证和数值模拟表明, 如果扩散策略依赖于资源, 即使一个物种的扩散速度较慢, 尽管在不同条件下可能会发生竞争排斥, 但共存也是可能的。这与随机分散的结果不同, 较慢的扩散器总是消灭其快速的竞争对手。在数值模拟的支持下, 我们的分析结果表明, 依赖资源的分散策略对种群动态和进化过程具有重要的影响。

## **Gross-Pitaeski Equation, Adler-Moser Polynomials and Kadomtsev-Petviashvili Lump**

**Juncheng Wei**

University of British Columbia

jcwei@math.ubc.ca

We consider the traveling wave solutions to the Gross-Pitaeski equation with speed  $c$ . We show that when the speed  $c$  is small the existence of higher energy solutions is reduced to the study of Adler-Moser polynomials, while when  $c$  is close to the sound speed  $\sqrt{2}$ , there is a deep connection with KP-I equation.

## **Planar polynomial differential systems with a global center**

**Dongmei Xiao**

Shanghai Jiaotong University

xiaodm@sjtu.edu.cn

In this talk, we will introduce the characterization on planar polynomial differential systems having a global center, that is, every orbit of the system is a periodic orbit in the plane. Further, we will give algebraic sufficient and necessary conditions for potential systems and Lienard systems which have a global center, respectively. Last we will discuss some related problems. This is based on a joint work with Hongjin He.

## **Quasi-Stationary Distributions**

**Yingfei Yi**

University of Alberta and Jilin University

yingfei@ualberta.ca

Quasi-stationary distributions (QSDs), representing transient states of a diffusion process admitting an extinction state, arise frequently in applications especially in chemical reactions and population dynamics, but there are only limited rigorous studies on the subject. This talk will present some new results on the existence, uniqueness, and convergence of QSDs along with their connections to the spectrum of the Fokker-Planck operator. Applications to

cooperative, competitive, and predator-prey Lotka-Volterra systems will be discussed.

## **Global asymptotic stability in a delay mosquito population suppression model**

**Jianshe Yu**

Guangzhou University

jsyu@gzhu.edu.cn

Based on the idea that only those sexually active sterile mosquitoes are included in the modeling process, we study the dynamics of the interactive wild and sterile mosquito model with time delay, which consists of three sub-equations. Due to the fact that the maturation period of sterile mosquitoes bred in the lab or mosquito factories is the same time period of wild adults mature from larvae, we particularly assume that the waiting period for two consecutive releases of sterile mosquitoes equals the maturation period of wild mosquitoes, as a new practical sterile mosquito release strategy. We first ingeniously solve the delay model with the initial functions that are solutions of the corresponding equation without delay, referred to as "good" solutions. Using "good" solution control method, we obtain sufficient and necessary conditions for the trivial solution and a unique periodic solution of the delay model to be globally asymptotically stable, respectively. We provide a numerical example to demonstrate the model dynamics and brief discussions of our findings as well. We are confident that the results obtained and the methods used in this paper can be applied to other similar situations where no existing results and methods are available.

## **Nonlinear Dynamics in Modeling Sea Lice with Stage Structure**

**Yuan Yuan**

Memorial University of Newfoundland

yyuan@mun.ca

Sea lice infection is one of the major threats in the marine fishery, especially for farmed salmon. In this work, we propose two mathematical models, one is for the growth of sea lice with a three-stage structure: non-infectious immature, infectious immature and adults where the level of non-infectious immature development depends on the size of the adult population; another is for the control of sea lice with the predator-prey interaction between cleaner fish and sea lice. By using mathematical techniques and an appropriate change of variables, we first describe the nonlinear dynamics by a system of partial differential equations, then transform it into a system of delay differential equations with constant delay. We provide two important indexes: the adult

reproduction number  $R_s$  for sea lice and the net reproductive number of cleaner fish  $R_f$ , address the global dynamics relating to  $R_s$  and  $R_f$  theoretically, including the global/local stability of the equilibria, uniform persistence and possible Hopf bifurcation. Numerical simulations are provided to confirm the theoretical results.

## **Predator-Prey models with Sigmoid functional response**

**Xiang Zhang**

Shanghai Jiaotong University

xzhang@sjtu.edu.cn

In this talk we introduce our results on global dynamics of the predator-prey model with the Sigmoid functional response. The main results are global stability of the positive equilibrium, order of weak focus, existence of limit cycles, canard explosion and relaxation oscillation. The main tools are qualitative analysis and geometric singular perturbation.

## **Propagation Dynamics of Reaction and Diffusion Equations in a Time-heterogeneous Shifting Environment**

**Xiaoqiang Zhao**

Memorial University of Newfoundland

zhao@mun.ca

In this talk, I will report our recent research on the propagation dynamics of a large class of nonautonomous reaction-diffusion equations with the time-dependent shifting speed having a uniform mean  $c$ . Under the assumption that in two directions of the spatial variable there are two limiting equations with one admitting a spreading speed  $c^*$  and the other being asymptotic to annihilation, we show that the solutions with compactly supported initial data go to zero eventually when  $c$  is less than or equal to  $-c^*$ , the leftward spreading speed is  $-c^*$  when  $c$  is greater than  $-c^*$ , and the rightward spreading speed is  $c$  and  $c^*$  when  $c$  is in the interval  $(-c^*, c^*)$  and  $c$  is greater than or equal to  $c^*$ , respectively. We also establish the existence, uniqueness and nonexistence of the forced traveling wave in terms of the sign of  $c - c^*$ . This talk is based on a joint work with Dr. Lei Zhang.

## **Bifurcation of nilpotent singularities and Hilbert's 16th problem for quadratic vector fields**

**Huaiping Zhu**

York University

huaiping@yorku.ca

In this talk, I will introduce the classification of nilpotent singularities, their normal forms and bifurcations. Then I will explain the finiteness part of the Hilbert's 16th problem for quadratic vector fields and report some recent results on the problem. We showed that the number of limit cycles which can be bifurcated from the graphics with nilpotent singularity for quadratic vector fields is at most two.

## **Evolution and adaptation of anti-predation response of prey in a two-patchy environment**

**Xingfu Zou**

Western University

xzou@uwo.ca

When perceiving a risk from predators, a prey may respond by reducing its reproduction and decreasing or increasing (depending on the species) its mobility. We formulate a patch model to investigate the aforementioned fear effect which is indirect, in contrast to the predation as a direct effect, on the prey population. We consider not only cost but also benefit of anti-predation response of the prey, and explore their trade-offs as well as the impact of the fear effect mediated dispersals of the prey. In the case of constant response level, if there is no dispersal and for some given response functions, the model indicates the existence of an evolutionary stable strategy (ESS) which is also a convergence stable strategy (CSS) for the response level; and if there is dispersal, the analysis of the model shows that it will enhance the co-persistence of the prey on both patches. Considering the trait as another variable, we continue to study the evolution of anti-predation strategy for the model with dispersal, which leads to a three-dimensional system of ordinary differential equations. We perform some numerical simulations, which demonstrate global convergence to a positive equilibrium with the response level evolving toward a positive constant level, implying the existence of an optimal anti-predation response level. Interestingly, it is observed that this optimal response level may not agree with the ESS. This is a joint work with Ao Li.